Versatile Kirchhoff Code for Aeroacoustic Predictions

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Introduction

THE aerodynamic noise generated from fluids in motion has been identified as the dominant noise mechanism for aircraft and rotorcraft noise. Fortunately, with the fast development of high-speed computers, computational fluid dynamics (CFD) is now frequently employed to solve the near-field aerodynamics. Once the sound source is predicted, several approaches can be used to describe its propagation. The obvious strategy is to extend the computational domain for the full, nonlinear Navier–Stokes equations far enough to include the location where the sound is to be calculated. However, if the objective is to calculate the far-field sound, this direct approach requires prohibitive computer storage and leads to unrealistic CPU times. The best solution seems to be the separation of the computation into two domains, one describing the generation of sound, and the other describing the propagation of sound.

There are several alternatives to describing the sound propagation once the source has been identified, the first of which is the acoustic analogy. In this method the governing Navier–Stokes equations are rearranged to be in wave-type form. The far-field sound pressure is then given in terms of a volume integral over the domain containing the sound source. However, computation of the volume integrals leads to storage problems. Another method is to use a CFD near-field solution plus a linearized Euler's equation. This method starts from the numerical calculation of the nonlinear near field and midfield, whereas the far field is found from a linearized Euler solution. This method seems to have very good potential; however, the solution still has to be found in the far field numerically. Thus, far-field mesh spacing, diffusion, and dispersion errors have to be adequately resolved.

The final alternative is the Kirchhoff method. The method consists of the calculation of the nonlinear near field and midfield, usually numerically, with the far-field solutions found from a linear Kirchhoff formulation evaluated on a surface S surrounding the nonlinear field. The surface S is assumed to include all of the nonlinear flow effects and noise sources. The linear wave equation is assumed valid outside the control surface. The separation of the problem into linear and nonlinear regions allows the use of the most appropriate numerical methodology for each. The advantage of this method is that the surface integrals and the first derivatives needed can be evaluated more easily than the volume integrals and the second derivatives needed for the evaluation of the quadrupole terms when acoustic analogy is used. The method is simple and accurate and accounts for the nonlinear quadrupole noise in the far field.

This idea of matching between a nonlinear aerodynamic near field and a linear acoustic far field was first proposed by Hawkings.[1] The use of the Kirchhoff method has increased substantially the last 5–10 years, because of the development of reliable CFD methods that can be used for the evaluation of the near field. It has been used to study various aeroacoustic problems, such as propeller noise, high-speed compressibility noise, blade–vortex interactions, jet noise, ducted fan noise, etc. A review of the uses of the Kirchhoff method in computational aeroacoustics is given by Lyrintzis.[2]

For the Kirchhoff method to become a useful general aeroacoustics tool, a versatile Kirchhoff methodology is needed. This is the objective of this paper. A retarded time calculation has been traditionally used for Kirchhoff subroutines. A new approach is developed herein, which accumulates signals time marched from each surface element to an observer; thus it avoids the retarded time calculation. The approach was motivated by a similar idea that was proposed by Brentner[3] for the evaluation of integrals used in the acoustic analogy method. In this approach, the final overall observer acoustic signal is found from the summation of the acoustic signal radiated from each source element of the Kirchhoff surface during the same source time. Computer memory requirements are reduced dramatically and the algorithm is inherently parallel. Note that the same approach is also used in Ref. 4 for the simple case of rectilinear Kirchhoff surface motion. The proposed algorithm is versatile, allows memory segmentation, and provides natural parallelization. Thus, a versatile code is developed to calculate the far-field noise from inputs supplied by any aerodynamic near- and/or midfield code. An extended version of this work can also be found in Refs. 5 and 6.

Kirchhoff Method

The Kirchhoff formula was first published in 1882. Although primarily used in the theory of diffraction of light and in other electromagnetic problems, it has many applications in studies of acoustic wave propagation. The classical Kirchhoff formulation is limited to a stationary surface. Morgans[7] derived a Kirchhoff formula for a moving surface (i.e., the interior region of an expanding sphere) using the Green function approach.

Generalized functions can also be used for the derivation of an extended Kirchhoff formulation. Farassat and Myers[8] derived an extended Kirchhoff formulation for a moving, deformable, piecewise smooth surface. Most current Kirchhoff codes use this retarded time formulation. The formulation gives the pressure signal of an observer in the stationary coordinate system as a function of the surface integral over the moving control surface S of the pressure, the normal derivative of the pressure, and the time derivative of the pressure:

$$p(X_*, t_*) = \frac{1}{4\pi} \iint_S \left[\frac{E_1}{r(1-M_r)} + \frac{E_2 p}{r^2(1-M_r)} \right]_{\tau^*} \mathrm{d}S \qquad (1)$$

where

$$E_1 = \left(M_n^2 - 1\right) p_n + M_n \boldsymbol{M}_t \cdot \nabla_2 p - a_\infty^{-1} M_n \dot{p}$$
$$+ \frac{a_\infty^{-1}}{(1-M_r)} [(\dot{n}_r - \dot{M}_n - \dot{n}_M) p + (\cos\theta - M_n)\dot{p}]$$
$$+ \frac{a_\infty^{-1}}{(1-M_r)^2} [\dot{M}_r (\cos\theta - M_n) p]$$
$$E_2 = \frac{1 - M^2}{(1-M_r)^2} (\cos\theta - M_n) \qquad (2)$$

where $\boldsymbol{M}$ and $\boldsymbol{n}$ are the Mach number and the unit normal to the Kirchhoff surface. Here the dot over M and n denotes the source time τ derivative; t_* is the observer time; $\boldsymbol{X}_*$ is the observer position; $\boldsymbol{X}$ is the surface (source) position; a_∞ is the speed of sound outside the Kirchhoff surface; $\boldsymbol{r}$ is $\boldsymbol{X}_* - \boldsymbol{X}$; r is the distance from observer to source, ($|\boldsymbol{X}_* - \boldsymbol{X}|$); and $\cos\theta$ is the directivity parameter, $\hat{\boldsymbol{r}} \cdot \boldsymbol{n}$, where $\hat{\boldsymbol{r}} = \boldsymbol{r}/r$. The brackets in Eq. (1) denote evaluation over the retarded (emission) time τ^*, which is the solution of the equation

$$\tau - t_* + |X_* - X(\tau)|/a_\infty = 0 \qquad (3)$$

In addition, the following definitions are introduced:

$$\dot{M}_r = \dot{\boldsymbol{M}} \cdot \hat{\boldsymbol{r}}, \qquad \dot{n}_r = \dot{\boldsymbol{n}} \cdot \hat{\boldsymbol{r}}$$
$$\dot{M}_n = \dot{\boldsymbol{M}} \cdot \boldsymbol{n}, \qquad \dot{n}_M = \dot{\boldsymbol{n}} \cdot \boldsymbol{M} \qquad (4)$$

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The retarded time equation (3) has a unique solution when the surface moves subsonically. A Newton–Raphson method is utilized to solve this nonlinear equation. However, it is difficult to write a versatile code for various mesh topologies used by current CFD codes, including unstructured grids, based on this approach. In addition, the codes sometimes require significant memory.

To overcome the limitations just stated, a new approach is developed that accumulates signals time matched from each surface element to an observer; thus it avoids the retarded time calculation. Computer memory requirements are reduced dramatically and the algorithm is inherently parallel.

In this approach, the final overall observer acoustic signal is found from the summation of the acoustic signal radiated from each source element of the Kirchhoff surface during the same source time. The observer time is a straightforward calculation using Eq. (3). For each surface element time is marched forward from the source (emission) to the observer time. Since a different surface element will result in a different observer time, interpolation techniques are required when the integration is performed to obtain the overall acoustic signal at the observer position. Both linear interpolation and spline subroutines have been used herein. For high frequencies a digital filter may be used to increase accuracy.[9] Additional care is taken for periodic signals to ensure periodicity.

Farassat and Myers'[8] Kirchhoff equation (1) is used for the general Kirchhoff aeroacoustic (GKA) code developed herein. A standard input format to the GKA code is used for all aeroacoustics problems. The input includes pressure, its time, and normal derivatives, as well as the motion of each Kirchhoff surface element. Since no formal surface integration is performed, no grid topology is needed as input for the Kirchhoff subroutine. This enhances the versatility of the code, so that it can be used with any finite difference, finite element, or finite volume CFD code with structured or unstructured grids. The computation can be easily split into several subsets of source elements, which allows memory segmentation and provides natural parallelization with no communication cost except for the final summation. Parallelization could become an important issue when many (e.g., 10^6) surface elements are used in the CFD solver. Since the retarded time equation has multiple roots for a supersonically moving surface, the proposed approach could be very valuable for such case, although interpolation errors may increase.

The penalties associated with the versatility of the GKA code are that more hard disk memory is required and the code is less efficient for simple cases such as a stationary point source because the interface was designed for the general and complicated cases, such as rotor noise from an elastic blade with full rotor dynamics.

Results and Discussion

To validate the GKA code a point source input problem is considered. As a first test an observer point was placed inside the Kirchhoff surface and the result was very close to zero, as expected. For observer points outside the Kirchhoff surface the overall agreement with the analytical solution was excellent, i.e., less than 0.1% relative error from the analytical solution for 50 points per period and 20 points per wavelength.

The full potential rotor (FPR) code[10] is used for validation because the study of rotorcraft impulsive noise prediction has been already done with the traditional retarded time approach.[11] Other tests with less complicated Kirchhoff surface motions are actually easier to implement. The CFD data from FPR is rotating. The rotating Kirchhoff method[11] is chosen where the Kirchhoff surface rotates with the blade (see Fig. 1). The rotating surface case was chosen because it offers the most difficult test. However, note that the code can also be used with a nonrotating Kirchhoff surface, depending on the source data provided by CFD codes. In such a case an extra transformation from rotating frame to nonrotating frame is needed for the CFD input data.

The numerical studies were performed on an SGI ONYX workstation. The results shown are for a nonlifting rotor with the NACA 0012 airfoil section shown in Fig. 1; aspect ratios of 13.7 for high-speed impulsive (HSI) and 7.125 for the blade–vortex interaction (BVI) were used to match experiments (the line vortex is parallel to the freestream). The FPR code with an $80 \times 25 \times 25$ mesh is used

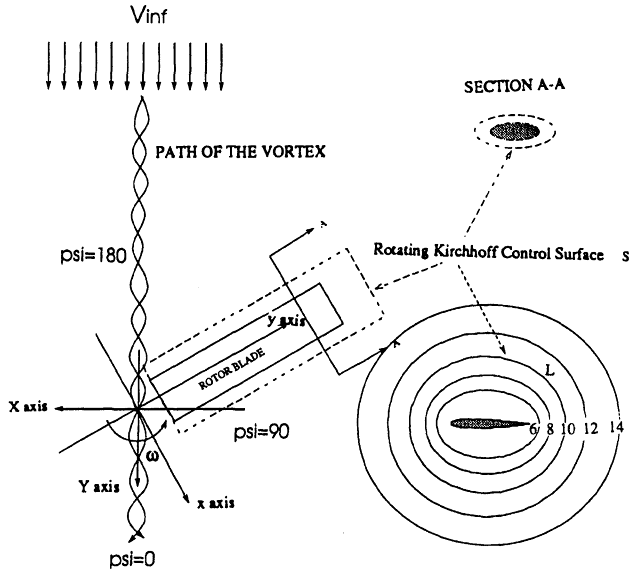

Fig. 1 Rotating blade, parallel BVI, and rotating Kirchhoff surface.

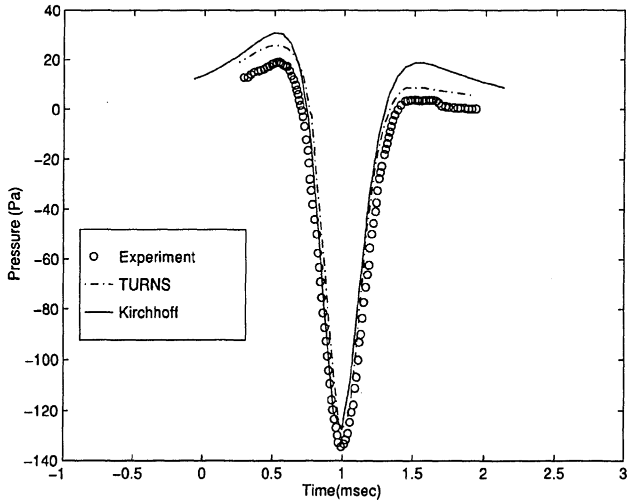

Fig. 2 Comparison of predicted HSI noise with experiments[14] and results[12] for scaled UH-1H rotor with hover $M_{\rm tip} = 0.85$, with the observer at 3.09 rotor radii in the rotor plane.

with a 0.25-deg rotor azimuth per time step for 1440 time steps, which takes about 7 h of CPU time on our SGI workstation. For the hover cases the CPU time is much less because the steady FPR solution can be used. The computation of rotor far-field acoustic signals usually takes less than 5 min of CPU time for far-field acoustic results radiated from one blade revolution.

Since the rotating Kirchhoff method assumes that linear equations are valid outside the closed rotating control (Kirchhoff) surface S, S must be chosen large enough to include the region of all nonlinear behavior. However, the accuracy of the numerical solution is limited to the region immediately surrounding the moving blade because of the increase of mesh spacing with distance in the FPR code. Thus a judicious choice of S is required for the effectiveness of the Kirchhoff method. Surfaces S from $L = 6$ to 20 are chosen (based on our experience), where L is the grid index in the direction normal to the body shown in Fig. 1. Higher Mach numbers require larger surfaces because the persistence in the nonlinear regions are larger. A grid index of $L = 11$–14, i.e., about one to two chords away from the airfoil (see Fig. 1 for details of the shape), and $K = 20$, i.e., about one-half to one chord away from the tip (K is the grid index of the Kirchhoff surface in the spanwise direction) are used in the subsequent calculations. A closed Kirchhoff control surface S (Fig. 1) consists of three surfaces: one is a cylinder-like surface warped around the rotating blade, and the other two surfaces are two base surfaces on the tip and hub of the cylindrical surface. However, the effect from the hub base surface is insignificant, and thus only the cylindrical surface and tip base surface are considered herein. Because a retarded time formulation is used [Eq. (1)], the tip base surface must be inside the sonic cylinder.

Figure 2 shows the computed in-plane far-field pressure for $M = 0.85$ for HSI hover noise for the one-seventh scale model of the UH-1H two-bladed rotor. The rotor has a NACA 0012 airfoil with a radius of 1.045 m with a chord of 7.62 cm. The figure shows results from Ref. 12, i.e., straight CFD results from the transonic unsteady rotor Navier–Stokes (TURNS) code[13] with no Kirchhoff extensions, and experimental results.[14] The observer is at a distance of 3.09 rotor radii in the rotation plane. The results from the GKA code are also shown. Very good agreement was found, although GKA uses as input CFD results from the less accurate full potential FPR code. Several other tip Mach numbers were studied, as well as BVI noise; details are shown in Refs. 5 and 6.

Concluding Remarks

A new versatile Kirchhoff code was developed based on the accumulation of signals on an observer's position avoiding surface integration at the retarded time. The final overall observer acoustic signal is found from the summation of the acoustic signal radiated from each source element of the Kirchhoff surface during the same source time. A standard input format is used for all aeroacoustics problems. No detailed grid topology is needed as input for the Kirchhoff subroutine. Thus the code can be used with any finite difference, finite element, or finite volume CFD code with structured or unstructured grid. The computation can be easily split into several subsets of source elements, which allows memory segmentation and provides natural parallelization with no communication cost except for the final summation. The code was validated using a point source. Very good agreement with experiments was found for rotor HSI and BVI noise prediction using CFD input from the full potential FPR code.

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Discrete Probability Function Method for the Calculation of Turbulent Particle Dispersion

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Introduction

DISPERSION of liquid drops in spray combustion systems is a critical parameter for high combustion efficiency. In modern lean-premixed-prevaporized (LPP) low NO_x combustors, the design of premixers to achieve complete drop evaporation and fuel-air mixing prior to combustion is crucial to the success of such systems. Drop dispersion determines the residence time of the drop in the premixer, prescribes local boundary conditions for drop heat and mass transfer processes, and enhances interface area and fuel vapor concentration gradients to promote faster evaporation and molecular mixing. Thus, the performance of premixers in LPP combustion systems seems to be dispersion limited.

Most dispersion calculations involve separated flow trajectory models,[1] which require the simulation of a large number of particle trajectories to obtain statistically significant results and to reduce shot noise. Even with a large number of computational parcels, events with lower probability may not be adequately represented. These events can be important in LPP combustion systems that depend on burning everywhere at overall lean conditions. For example, excursions in the equivalence ratio due to the presence of a relatively small number of large drops can lead to large variations in global NO_x emissions. These events, though rare, are responsible for almost all of the NO formation. In this Note, a discrete probability density (DPF) method is applied to drop dispersion calculations to provide accurate simulations with reduced statistical noise and to simulate the occurrence of rare events.

Theoretical Methods

Particle Motion

The simplified equation for particle motion neglecting forces due to pressure gradients, virtual mass, and the Basset effect is linearized as follows[2]:

$$\boldsymbol{u}_p \approx \boldsymbol{u}_{p0} \exp(-\Delta t/\tau) + (\boldsymbol{u}_g + \boldsymbol{g}\tau)[1 - \exp(-\Delta t/\tau)] \quad (1)$$

$$\Delta \boldsymbol{x}_p \approx \boldsymbol{u}_{p0}\tau[1 - \exp(-\Delta t/\tau)] + (\boldsymbol{u}_g + \boldsymbol{g}\tau) \times \{\Delta t - \tau[1 - \exp(-\Delta t/\tau)]\} \quad (2)$$

where τ is the particle relaxation time related to the Stokes number.[2]

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